XXII. *On the magnetism developed in copper and other substances during rotation. In a Letter from* Samuel Hunter Christie, *Esq. M. A. &c. to* J. F. W. Herschel, *Esq. Sec. R. S. Communicated by* J. F. W. Herschel, *Esq.*

Read June 16, 1825.

Dear Sir,

As you inform me that you are drawing up an account of your magnetical experiments, I send you a brief account of those which I have made: they may possibly bear upon some of the points which you have had under consideration; and in this case you will not be displeased at being able to compare independent results.

After having made experiments with a thin copper disk suspended over a horse-shoe magnet, similar to those which I witnessed at Mr. Babbage's, I made the following.

A disk of drawing paper was suspended by the finest brass wire (No. 37) over the horse-shoe magnet, with a paper screen between. A rapid rotation of the magnet (20 to 30 times per second) caused no rotation in the paper, but it occasionally dipped on the sides, as if attracted by the screen, which might be the effect of electricity excited in the screen by the friction of the air beneath it.

A disk of glass was similarly suspended over the magnet: no effect produced by the rotation.

A disk of mica was similarly suspended: no effect.

The horse-shoe magnet was replaced by two bar magnets, each 7.5 inches long, and weighing 3 oz. 16 dwt. each, placed horizontally parallel to each other, and having their poles of the same name contiguous. These produced quick rotation in a heavy disk of copper 6 inches in diameter, and suspended by a wire, No. 20.

A bar magnet 4 inches long, and having both its ends south poles, was made to revolve rapidly under a copper disk. The disk revolved in the same direction as the magnets.

The two bar magnets before mentioned were adjusted to the axis of rotation, so that their upper ends were at the distance of 5 inches from each other, and their lower ends 1.8 inch apart. They were first made to revolve rapidly under the copper disk with poles of the same name nearest to the disk, and then with poles of a contrary name: the times in which the several rotations of the disk took place were as nearly as possible the same in the two cases.

No. of Revolutions.	Poles of the same name nearest to the disk.		Poles of a contrary name nearest to the disk.	
	Screw.*	Unscrew.*	Screw.	Unscrew.
	Time.	Time.	Time.	Time.
1	15 sec.	15 sec.	15 sec.	15.4
2	21	21	21	21.5
3	26	26	26	26.3
4	30	30	30	30.0

In the first three, I could only remark the time to the nearest second, having no assistance. Should the times agree precisely, which I have very little doubt they would be found to do, the result would, I think, be singular. It would

* These expressions refer to the direction with respect to the spectator in which the rotation was performed.

show that the magnetism in the disk is instantaneously developed by one pole of the magnet, and as instantaneously destroyed, and a contrary magnetism developed by the contrary pole; or rather it would indicate, that the time during which the disk retained the induced magnetism was less than the time of half a revolution of the magnet.

The same two bar magnets were laid horizontally by the side of each other, $\frac{4}{10}$ inch a part. They were first made to revolve rapidly under the disk with their poles of the same name adjacent, and then with those of a contrary name adjacent.

No. of revolutions.	Poles of the same name adjacent.		Poles of a contrary name adjacent.	
	Screw.	Unscrew.	Screw.	Unscrew.
	Time.	Time.	Time.	Time.
	m. s.	m. s.	m. s.	m. s.
1	29	28	31	32
2	40	57	42	46
3	48	46	52	58
4	56	53	1 01	1 07
5	1 03	1 00	1 11	1 15
6	1 10	1 06	1 18	1 23
7	1 16	1 12	1 25	1 30
8	1 21	1 17	1 31	1 36
9	1 26	1 22	1 37	1 43
10	1 31	1 27	1 43	1 49

From these it appears that the effect was but little diminished by placing poles of a contrary name so close to each other.

The adjacent poles being of the same name, they were connected by a piece of soft iron $\frac{1}{8}$ inch thick, and $\frac{1}{2}$ inch wide. After $4\frac{1}{2}$ revolutions of the disk (screw), the torsion of the wire was equal to the force of the magnets, and the same was the case at $4\frac{3}{4}$ revolutions (unscrew). So that although the effects were greatly diminished by connecting the poles, they were by no means destroyed.

The magnets were now placed over each other, first with poles of a contrary name, and then with those of the same name contiguous.

No. of revolutions.	Poles of a contrary name contiguous.		No. of revolutions.	Poles of the same name contiguous.	
	Screw.	Unscrew.		Screw.	Unscrew.
	Time.	Time.		Time.	Time.
	m. s.	m. s.		s.	s.
1	1 48	1 32	1	21	21
2	3 20	2 40	2	30	29
$2\frac{1}{12}$	3 50	(Rev. $2\frac{1}{3}$) 3 45	3	36	34
			4	42	39
At $2\frac{1}{12}$ (screw) and $2\frac{1}{3}$ (unscrew) the torsion of the wire was equal to the force of the magnets.			5	47	44
			6	51	48

So that although the upper magnet was nearer to the disk, by its own thickness, than in the 4th experiment, the effect when poles of contrary name contiguous was not half what it was when they were connected by the iron.

A thick copper plate 8 inches in diameter and 1 inch thick, was placed on the axis of rapid rotation, its plane horizontal. A thin copper disk 4 inches diameter, and weighing 23.5 dwts. was very delicately suspended over it by a fine brass wire (No. 37), with a paper screen between the plate and the disk. The distance between the surfaces of the plate and disk $\frac{5}{10}$ inch. The plate being put in rapid rotation, no sensible effect was produced on the disk.

A bar magnet was placed on the screen under the disk: still no effect produced by the rotation.

A light needle, weight 42.5 grains, 6 inches long, on a pivot in a compass box, being placed over the plate, the rotation caused a deviation of 20°; but when a heavy needle, weighing 197 grains, and of the same length, was similarly placed over the plate, it immediately revolved rapidly with the plate.

A bar magnet, weighing 3 oz. 15 dwts. 19 grs. suspended by a wire, No. 20, revolved rapidly with the plate.

A horse-shoe magnet, weighing nearly a pound, and suspended by the same wire, revolved with the disk.

The following experiments were made with the view of ascertaining whether the effects increased nearly according to any power of the decrease of the distance.

A strong needle, 6 inches in length, weighing 197 grains, and vibrating 22 times in a minute, delicately suspended on an agate within a rim accurately graduated, was placed with its centre exactly over that of the copper plate, and being accurately adjusted, so that the distance between the centre of the copper and that of the needle was such as I required for the observation, the copper was made to revolve rapidly (always as nearly as possible 12 times per second), and when the needle became stationary, the direction of its south end (being that most convenient for observation) was noted. This was done with the copper revolving in both directions, "screw" and "unscrew." The direction of the south end of the needle was also observed before the rotation.

Distance.	4.0 in.	3.5 in.	3.0 in.	2.5 in.	2.0 in.	
Screw - -	1° 46′ W	3° 20′ W	6° 20′ W	14° 30′ W	29° 40′ W	Direction of south end of the needle.
Unscrew -	1 32 E	3 08 E	6 00 E	13 50 E	29 00 E	
Mean - -	1 39	3 14	6 10	14 10	29 20	

On diminishing the distance to 1.5 inch, the needle revolved with the plate, and very shortly so rapidly, that it had the appearance of an entire circle.

After this I replaced the needle by others which were lighter, letting every thing else remain the same, that is, the distance still 1.5 inch.

	Needle weighing 42.5 grs.	Needle weighing 25.5 grs.
Screw - -	24° 40′ W	10° 30′ W
Unscrew -	25 20 E	10 40 E

(I should mention that the needles were not at all neutralized).

From the latter observations, it is evident that the effect produced depends upon the intensity of the magnetism in the needle employed; and this I think proves clearly that the effect arises from the magnetism induced in the copper from the needle itself.

If we suppose the tang. of the deviation to vary as $\frac{1}{(\text{dist.})^n}$, then θ and θ' being two deviations at the distances d and d', we shall have $n = \frac{\text{log. tan. } \theta' - \text{log. tan. } \theta}{\text{log. } d - \text{log. } d.}$.

Computing n from this, by a comparison of every two observations we have the following values of n:

When distance is measured from centre of copper.	When distance is measured from surface of copper.
5.04	4.37
4.60	3.93
4.62	3.88
4.29	3.51
4.20	3.60
4.45	3.64
4.10	3.31
4.65	3.80
4.07	3.23
3.59	2.78
Mean 4.361	Mean 3.605

If we suppose that the poles of the needle are urged by forces in the direction of the motion of the copper, which being constant in the copper, would affect the needle reciprocally as the square of the distance; then these forces in the copper being derived from the needle itself, we must suppose that their intensity will vary also reciprocally as the square of the distance: so that the force on the needle arising from this mutual action, would vary reciprocally as the fourth power of the distance. Taking the mean between the mean values of n above, when the distance is measured from the centre of the copper and from its surface, would give the value of n for an intermediate point 3.983, which is as near to 4, supposing that such ought to be the value, as we could expect the observations to give.

The next experiments which I made were with the view of determining the law of force as regards the distance, when magnets act upon a copper disk. For this purpose I made use of the suspending wire as a balance of torsion. The results which I have obtained in this manner give a much less rapid diminution of the force, as the distance increases, than appears to take place when a thick copper plate acts upon a small magnet, as in the former experiments, which agrees with what you have mentioned as following from your results. The results obtained in the former case appear to indicate, that every particle in the copper urges the needle from the magnetic meridian with a force varying as $\frac{\text{vel. of particle}}{(\text{distance})^4}$, which law would arise from the magnetism in the needle developing the magnetism in the particles of copper, so that its intensity would vary as $\frac{1}{(\text{dist.})^2}$, and this magnetism again acting on the poles of the needle with a force varying as $\frac{1}{(\text{dist.})^2}$. Supposing this to be the case, if z is the distance of a lamina of copper from the plane of the needle, s the arc of a circle in this lamina at the distance r from the axis of rotation, R the radius of the copper cylinder, t its thickness, c the distance of its upper surface from the needle, and a the distance of the pole of the needle from its centre: then the whole force with which the cylinder urges the needle will be proportional to

$$\iiint \frac{r\,ds\,dr\,dz}{\left\{z^2 + (a-r)^2\right\}^2}$$

Although this may be integrable, the integral would be in so complicated a form, that it would be very ill suited for comparison with the results obtained from observation; but if we consider only the annulus of the copper immediately under the pole of the needle, which will be the most efficient part, we may readily make this comparison. For calling θ the deviation, we should have $\sin.\,\theta = \int \frac{dz}{z^4} \times$ const. or $\sin.\,\theta = \left(\frac{1}{c^3} - \frac{1}{(c+t)^3}\right) \times$ const.; and consequently $\frac{\sin.\,\theta}{\frac{1}{c^3} - \frac{1}{(c+t)^3}}$ = const.

From my experiments t being 1, I should obtain the following values of $\frac{\sin. \theta}{\frac{1}{c^3} - \frac{1}{(c+t)^3}}$.

C	θ	$\frac{\sin. \theta}{\frac{1}{c^3} - \frac{1}{(c+t)^3}}$	
	° ′		
3.5	1 39	2.3316	Mean 2.505
3.0	3 14	2.6341	
2.5	6 10	2.6409	
2.0	14 10	2.8144	
1.5	29 20	2.1040	

Although there is a considerable difference in the numbers, especially the last, yet as the parts whose action is not considered have here the greatest effect, and all the observations are liable to errors arising from the difficulty of making the copper revolve with the same velocity in all cases, I think the agreement is sufficiently near to indicate that the copper acts as I have supposed. A thick copper ring would be best adapted for obtaining results for comparison; and when I have leisure I propose making use of one.

For the purpose of determining the law according to which magnets act upon a copper disk at different distances, I suspended, successively, two copper disks over the bar magnets placed horizontally by the side of each other, with their poles of the same name adjacent. The magnets were made to revolve until the torsion of the wire caused the disk to return in the contrary direction, when I considered that the force of torsion would be double the force with which the magnets urged the disk. The time in which this took place was noted, and also the degree of torsion. After this the magnets were made to revolve again with the same velocity, and the torsion noted where the disk remained stationary by the action of the opposite forces of torsion and of the magnets. This was done at several distances; and those distances, between the magnets and the disk ascertained very accurately. In the observations with the disk which I have named A, the magnets were made to revolve with two different velocities; one of nearly 12 revolutions per second, the other of nearly 24 revolutions per second; but with the disk C the magnets always revolved with the velocity 24 revolutions per second, as I found that I could keep more steadily to this velocity than to the other. The length of the suspending wire (No. 22) was the same in both cases 34.25 inches. The thickness of the magnets is $\frac{1}{5}$ inch, so that I have added $\frac{1}{10}$ to the measured distances between the upper surface of the magnets and the copper, to reduce them to the distances between the plane of the copper and a horizontal plane passing through the axes of the magnets. The following tables contain the results.

Disk A, *weight* = 1305 *grains.*

Distance.	Screw.				Unscrew.			
	Unscrews.		Arc of torsion = force.		Screws.		Arc of torsion = force.	
	Arc.	Time.	Vel. 12.	Vel. 24.	Arc.	Time.	Vel. 12.	Vel. 24.
	°		°	°	°	m. s.	°	°
0.6	1330	Not obs.	760	1870	1160	1 09	700	1710
1.1	480	1m 10s	270	656	455	1 12	250	604
1.6	275	1 10	118	270	260	1 09	95	236
2.1	135	1 10	60	142	110	1 07	48	120
2.6	80	1 06	44	72	78	1 12	36	56

Disk C, *weight* = 2724 *grains.*

Distance.	Screw.				Unscrew.			
	Unscrews.		Arc of torsion = force.		Screws.		Arc of torsion = force.	
	Arc.	Time.	Vel. 12.	Vel. 24.	Arc.	Time.	Vel. 12.	Vel. 24.
	°	m. s.		°	°	m. s.		°
0.6	7270	1 44		3642	7750	1 43		3874
1.1	3465	1 42		1770	3380	1 40		1680
1.6	1670	1 40		834	1456	1 40		726
2.1	700	1 39		347	680	1 39		354
2.6	308	1 38		184	320	1 39		180

It is evident from these results, that the force with which the magnets urge the disk, as the distance increases, decreases much less rapidly than in the case of the copper plate revolving. If we suppose it to vary as $\frac{1}{\text{dist.}^n}$, then calling c and c' two distances and T and T′ the corresponding torsions, which are equal to the forces of the magnets, $n = \frac{\log. T - \log. T'}{\log. c' - \log. c}$.

Comparing the preceding results, the several values of n will be,

	Disk A.	Disk C.
Values of n	1.723	1.285
	1.995	1.556
	2.087	1.864
	2.271	2.065
	2.436	2.118
	2.429	2.406
	2.658	2.614
	2.420	2.803
	2.831	2.998
	3.354	3.246

These differ too widely from each other for us to suppose that the force varies as any exact power of the distance;

but the approximation is evidently towards the inverse square.

With regard to the forces with which different disks are urged at the same distance, they appear to be very accurately proportional to the weights of the disks when their distances from the magnets are small; but as the distances are increased, the forces appear to increase in a greater ratio than that of the weights of the disks.

Distance	.6	1.1	1.6	2.1	2.6	
$\frac{\text{Torsion}}{\text{Weight}} =$	1.372	.483	.194	.100	.049	Disk A.
$\frac{\text{Torsion}}{\text{Weight}}$	1.380	.633	.286	.134	.067	Disk B.

As it was only by a rough estimate, that I considered the velocity with which the magnets revolved under the disk A, was double in one case of what it was in the other, I would not, from these observations, pretend to determine the ratio of the forces as depending upon the velocities, but I should have little doubt that they are proportional.

From these experiments it appears, that the time in which the disk begins to return, by the torsion of the wire, is the same at all distances; and from another experiment it appeared to be independent of the velocity of rotation. This ought to be the case, the force accelerating the disk being constant; and the retarding force, the torsion, varying as the distance from a fixed point.

I fear that I have trespassed too long on your time by this account of the experiments which I have made, but had no idea of rendering it so long when I began. I shall be happy if any of these experiments throw any light upon the subject; and I beg you will make whatever use you think proper of them, and likewise of the account I before sent you.*

I am, dear Sir,

very truly your's,

S. H. CHRISTIE.

* In a former letter, dated May 11: the experiments related in which are embodied in this communication. (H.)

Royal Military Academy,
12th June, 1825.